1.

実数 x, y の関数 $f(x) = x^4 + 4x^3 - 12x^2$, $g(y) = y^2$ について、**問 (i)**∼**(vi)** に答えよ。解答は解答用紙の所定欄に日本語で記すこと。

(i) $f'(x)$ と $g'(y)$ を求めよ。

(ii) 方程式 $f(x) = 0$ の解を求めよ。

(iii) 関数 $f(x)$ のグラフをかけ。

(iv) $\displaystyle\int_{-2}^{0} \Big|f(x)\Big| \mathrm{d}x$ を求めよ。また、これは何を求めていることに対応するか述べよ。

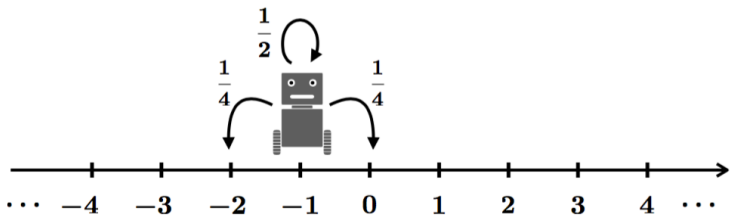
(v) ベクトル $\overrightarrow{v} = (x, y)$, $\overrightarrow{d} = (f'(x), g'(y))$ とする。ベクトル $\overrightarrow{w} = \overrightarrow{v} - \overrightarrow{d}$ を x と y を用いて表せ。

(vi) $\overrightarrow{v}$ と $\overrightarrow{w}$ のなす角を θ とする。ただし、$0 \leq \theta < \pi$ とする。$x = 1, y = 1$ のときの $\tan\theta$ の値を求めよ。

2.

問 (i), (ii) に答えよ。解答は解答用紙の所定欄に日本語で記すこと。導出過程も簡潔に記すこと。

(i) Let us assume a robot moving every second on the real line (see picture below). Let the probability of moving right $(+1)$ and left (-1) be $\frac{1}{4}$ each, and the probability of standing still $(+0)$ be $\frac{1}{2}$. At time $t = 0$, the robot is at position $x = 0$. Calculate the probability that the robot is at position $x = 0$ at time $t = 2$.



(ii) There are three water tanks A, B, and C. The water tanks are connected with each other through pipes. Each 1 minute, some fraction of the water is flowing from one tank to another. The fraction of water flowing from one tank to another in 1 minute is shown in the table below:

Table: Fraction of water flowing from one tank to another

Direction of water flow	A → B	A → C	B → A	B → C	C → A	C → B
Fraction of water	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{5}{6}$	0

Let us assume that the water flow ends within one minute, and furthermore, that the amount of water flowing during one minute from one tank to another is based on the amount of water at beginning of that minute. For example, assume that at a certain time (the beginning of one minute), tank A contains 100 liters. Then by the end of one minute tank B receives 25 liters, and tank C receives 75 liters from tank A.

Given some initial amount of water in each tank, the water flow described above repeats each minute. After some sufficient number of minutes have passed, the amount of water in each tank stabilizes (that is, there is no more change in the amount of water before and after one additional minute). Determine the proportion of water in each tank after stabilization (that is, the ratio of water in tank A : tank B : tank C).

3.

次の文を読み、**問 (i)**∼**(iv)** に答えよ。解答は解答用紙の所定欄に日本語で記すこと。

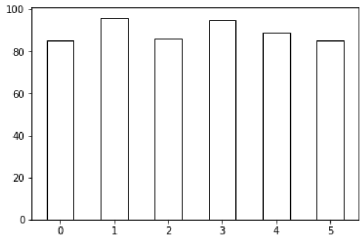
第二次大戦中、ロンドンはドイツ軍のミサイルによる爆撃にさらされた。ミサイルの着弾点の分布は、一部の狭い領域に偏って集中しているように見え、「そのような領域はドイツに狙われているのだ」とも考えられた。

南ロンドン地区における 537 回の着弾データを用いて、実際に特定の領域に狙いを定めてミサイルが発射されていたのかを統計学的に検証しよう。南ロンドン地区を 576 個の正方形区画に分割して、各区画毎に何回の着弾があったのかを数え上げた結果を集計したものが表 1 である。なお、ここで示した着弾回数のデータは Clarke, R. D. Journal of the Institute of Actuaries, Vol. 72, Issue 3, p.481 (1946) からの引用である。

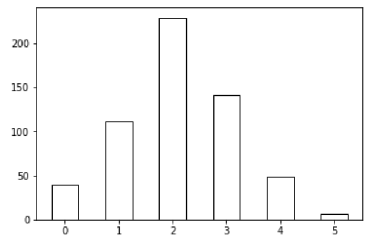
表 1: 区画内に着弾した回数の頻度分布

一区画における着弾回数	0 回	1 回	2 回	3 回	4 回	5 回以上
そのような区画の数 (度数)	229	211	93	35	7	1

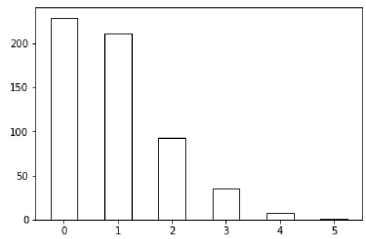
(i) 表 1 の度数分布表 (ヒストグラム) は次の (ア), (イ), (ウ) のうちどれに一番近いかを答えよ。



(ア)



(イ)



(ウ)

(ii) 一区画に着弾した回数の平均 λ を計算し、有効数字 2 桁で答えよ。

(iii) 一区画に着弾した回数の分散 σ^2 を計算し、有効数字 2 桁で答えよ。途中経過も簡潔に記すこと。

(iv) 表 1 の度数分布は、着弾回数を確率変数 X として、$X = k$ となる確率が

$$P(X = k) = \frac{e^{-\lambda}}{k!}\lambda^k$$

となるポアソン分布でよく表されることがわかった。ここで e は自然対数の底である。実際、(i), (ii) で計算したように、平均と分散の値がおよそ一致している。「特定領域に狙いを定めてミサイルが発射された説」について言えることは何か。

4.

次の問に答えよ。解答は解答用紙の所定欄に日本語で記すこと。導出過程も簡潔に記すこと。

Rikkyo baseball coaches are selecting the starting members for the next national game. Each selected player belongs to a different school. The Rikkyo University Newspaper reporters are keen to know the result of the selection, but the coaches only give them the following information.

1. Ken and the third baseman represent each school in social science (management and economics). Ken and the third baseman are different persons.
2. Tanaka and the pitcher represent humanities (arts and intercultural communication). Tanaka and the pitcher are different persons.
3. The catcher is Taro.
4. The captain Yota Sato is not in school of science nor is the third baseman. Yota Sato and the third baseman are different persons.
5. The first baseman is Kato.
6. Ito is in management and Jiro is in arts. Ito and Jiro are different persons.

List of first names: Ken, Taro, Jiro, Yota, Syo
List of last names: Sato, Tanaka, Ito, Kato, Yamada
List of schools: management, economics, arts, intercultural communication, science
List of positions: pitcher, catcher, first baseman, third baseman, shortstop

Fill in the list of five selected members with correct combinations of first names, last names, schools, and positions.

5.

次のような事案について論ぜよ。解答用紙の所定欄 (1 行は 30 字程度) に 300 字程度で日本語で記すこと。

In 2002, the crime prevention department (CPD) developed, in secret, "Criminal-Finder", an AI face recognition system for automatically identifying wanted criminals. The department tested the system during a traditional fireworks festival with 1 million visitors. Criminal-Finder was trained with high-quality photographs from 50 different wanted criminals. Out of 100,000 visitors, which faces were well captured by the surveillance cameras, Criminal-Finder classified 20 visitors as wanted criminals with 99% confidence in real-time.

Alarmed by the CPD, the police officers tried to arrest those 20 visitors. However, the police officers found that 17 (out of the 20) visitors had been misclassified by Criminal-Finder, and arrested the remaining 3 persons. Finally, after some more investigation, it turned out that only 1 person out of the arrested 3 persons were actually wanted criminals.

On the other hand, 3000 police officers, which were patrolling during the festival, were able to arrest 3 more actual criminals that were missed by Criminal-Finder.

The Criminal-Finder experiment caused a big social issue after the details of the experiment were revealed.